Unit 3– Inverse Trig Functions, Trig Functions of Angles and More Trig Equations

More Solving Trigonometric Equations (covered in 7.4 of text, we are covering throughout.)

Multistep Equations: First isolate the trig function, then solve for the argument

1) Solve: $2\cos\left(x\right)-1=0; \quad 0\le x<2\pi$

2) Solve $\tan^2\left(x\right)-1=0$

Solving when there is an **expression in the argument**.
First solve for the argument, then the variable.

3) Solve: $\sin\left(x-3\right)-1=0;$

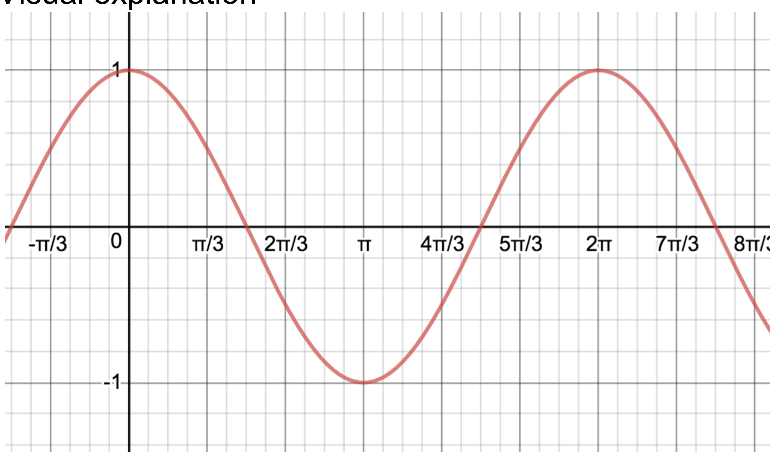
4) Solve $\tan\left(\frac{x}{3}\right)=\sqrt{3}$

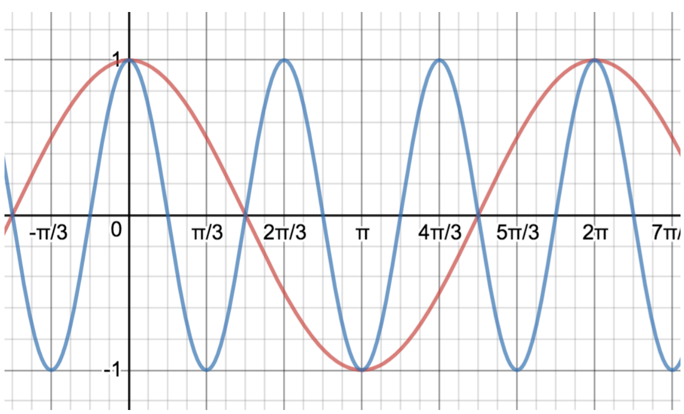
5) Find vertical asymptotes for $f(x)=\tan\left(2x\right)$

6) (a) Solve $\cos(3x)+1=0;$

(b) $\cos(3x)+1=0; \quad 0 \le x < 2\pi$

Visual explanation





5.5 Inverse Trig Functions

Solve: $0 \le x \le 2\pi$

$\sin(x) = \frac{1}{2}$ $\sin(x) = \frac{1}{3}$

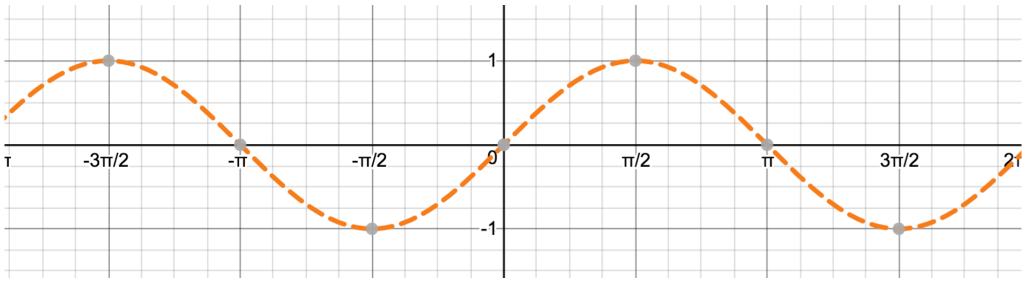
Review of Inverse Functions

Given $f(x) = \frac{1}{2}x^2 + 2\,; \quad x \ge 0$

- find $f^{-1}(x)$
- find the domain and range of $f(x)$ & $f^{-1}(x)$
- sketch a graph of $f(x)$ & $f^{-1}(x)$
- prove the functions are inverses by showing $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$

Inverse Sine Function

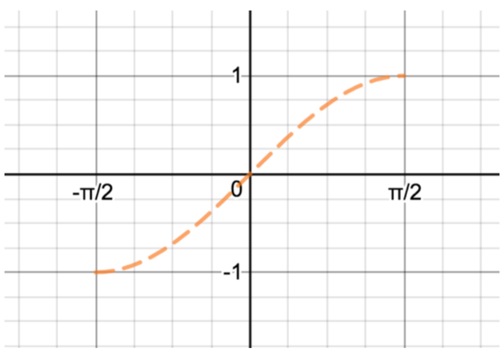
Does $g(x)=\sin(x)$ have an inverse? ______________________________________



What restriction would we need to make so that at least a piece of this function has an inverse?

Given $f(x)=\sin(x);$ ________________

1) Find $f^{-1}(x)$
2) Graph $f(x)$ and $f^{-1}(x)$.
3) Find the domain and range of $f(x)$ and $f^{-1}(x)$.



	$f(x)=\sin(x)$	$f^{-1}(x)=\sin^{-1}(x)$
Domain:		
Range:		

We define $y=\sin^{-1}(x)$ or $y=\arcsin(x)$ to mean $\begin{cases}\sin(y)=x \\ AND \\ -\frac{\pi}{2}\le y\le\frac{\pi}{2}\end{cases}$

Note: Both the input and output of this function are real numbers, but it is sometimes helpful to think in terms of angles.

that is let $\theta=\sin^{-1}(x)$ or $\theta=\arcsin(x)$ mean $\begin{cases}\sin(\theta)=x \\ AND \\ -\frac{\pi}{2}\le \theta\le\frac{\pi}{2}\end{cases}$

For example:

$$\sin\left(\frac{\pi}{6}\right)=\frac{1}{2} \qquad \sin^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{6}$$

$$\sin(angle)=number \qquad \sin^{-1}(number)=angle\ in\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$$

Finding exact values of the inverse sine function for special inputs: (like:______________________)

Ex: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Set $\theta=\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ and re-write according to the definition as ___________________

In words: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ *is the real number (or angle) in* $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ *whose sine (or y value on the unit circle) is* $\frac{\sqrt{3}}{2}$

Ex: $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

Since $y=\sin^{-1}(x)$ is a *function*, ________________________________

Since $f(x)=\sin(x)$ and $f^{-1}(x)=\sin^{-1}(x)$ are inverse functions,

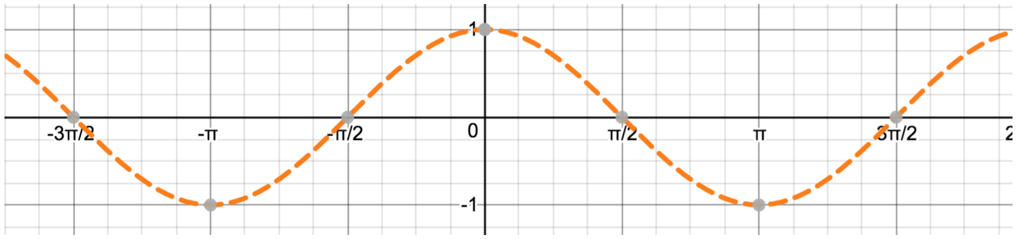
$f\left(f^{-1}(x)\right)=x$ ______________ $\qquad \sin\left(\sin^{-1}(x)\right)=x$ ______________

$f^{-1}\left(f(x)\right)=x$ ______________ $\qquad \sin^{-1}\left(\sin(x)\right)=x$ ______________

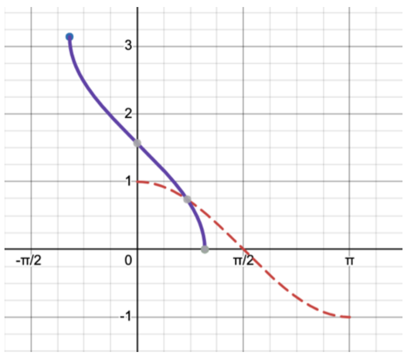
$\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right)$ $\qquad \sin^{-1}\left(\sin\left(\frac{\pi}{3}\right)\right)$

$\sin\left(\sin^{-1}3\right)$ $\qquad \sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right)$

Inverse Cosine Function



What restriction would we need to make so that at least a piece of this function has an inverse?



	$f(x)=\cos(x)$	$f^{-1}(x)=\cos^{-1}(x)$
Domain:		
Range:		

We define $y=\cos^{-1}(x)$ or $y=\arccos(x)$ to mean $\begin{cases}\cos(y)=x \\ AND \\ 0\le y\le \pi\end{cases}$

of angles. The development is similar to $\sin^{-1}(x)$, review as needed.

let $\theta=\cos^{-1}(x)$ or $\theta=\arccos(x)$ mean $\begin{cases}\cos(\theta)=x \\ AND \\ 0\le \theta\le \pi\end{cases}$

Finding exact values of the inverse cosine function for special inputs:

Ex: $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

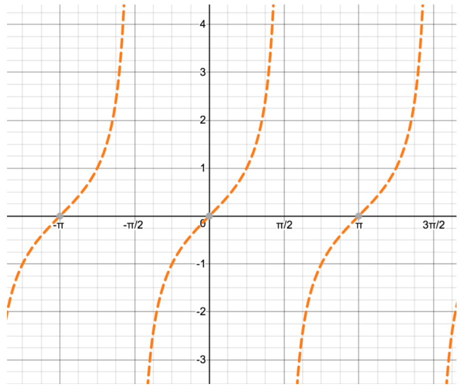
$\cos\left(\cos^{-1}(x)\right)=x$ ________________

$\cos^{-1}\left(\cos(x)\right)=x$ ________________

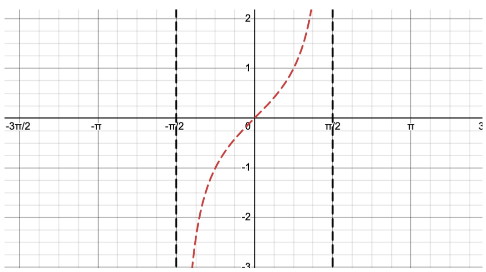
$\cos\left(\cos^{-1}0\right)$

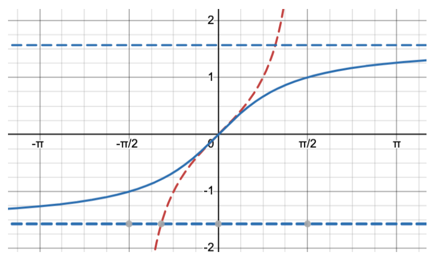
$\cos^{-1}\left(\cos\left(\frac{7\pi}{4}\right)\right)$

<u>Inverse Tangent Function</u>



What restriction would we need to make so that at least a piece of this function has an inverse?





$f(x) = \tan(x)$	$f^{-1}(x) = \tan^{-1}(x)$

Domain:

Range:

We define $y = \tan^{-1}(x)$ or $y = \arctan(x)$ to mean $\begin{cases} \tan(y) = x \\ AND \\ -\dfrac{\pi}{2} \le y \le \dfrac{\pi}{2} \end{cases}$

of angles. The development is similar to $\sin^{-1}(x)$

let $\theta = \tan^{-1}(x)$ or $\theta = \arctan(x)$ mean $\begin{cases} \tan(\theta) = x \\ AND \\ -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2} \end{cases}$

Finding exact values of the inverse tangent function for special inputs:

Ex: $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ $\tan^{-1}(-1)$

$\tan\left(\tan^{-1}(x)\right) = x$ ______________

$\tan^{-1}\left(\tan(x)\right) = x$ ______________

$\csc^{-1}(x)$, $\sec^{-1}(x)$, $\cot^{-1}(x)$, are mentioned in the book, but are not used often and will not be tested..

Solving Equations Using Inverse Trig Functions

Solve: $0 \le x \le 2\pi$

$\sin(x) = \frac{1}{2}$

$\sin(x) = \frac{1}{3}$

$\sin(x) = -\frac{\sqrt{2}}{2}$

$\sin(x) = -\frac{3}{4}$

$\cos(x) = \frac{3}{7}$

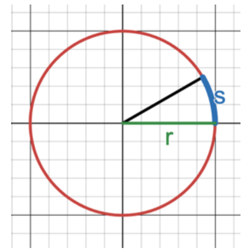
$\cos(x) = -0.42$

$\sin(x) = 3$

$\tan(x) = 4$

Worksheet 3: Solve Trig Equations using inverse

6.3 Trigonometric Functions of Angles

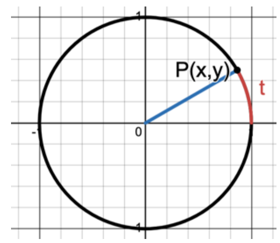


Previously we found that for an angle, θ, measured in radians, $s = r\theta$

So in a unit circle, we find ____________________

If we reconsider the way we defined the trig functions, for P(x,y) a point on the unit circle

$\sin(t) =$ _______ $\cos(t) =$ __________ ....etc



We can just as easily define them in terms of the angle, θ, independent of t. This doesn't change the way we compute them if we know the corresponding point of the unit circle.

$$\sin(\theta) = y \qquad \cos(\theta) = x$$

Now $\sin\left(\frac{\pi}{3}\right)$ can be thought of as the sine of the *real number* $\frac{\pi}{3}$, or sine of the *angle* $\frac{\pi}{3}$, which when viewed as an angle can also be measured in ________________

$$\sin\left(\frac{\pi}{3}\right) = \sin(\quad^\circ) = \_\_\_\_\_\_\_\_$$

Example: Find the following exactly

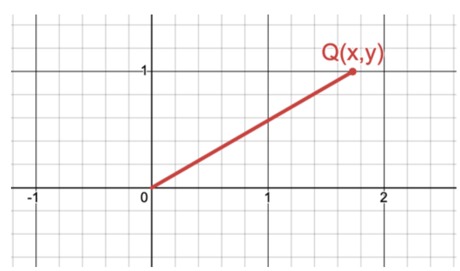
$\sin(120^\circ) =$ ______________ $\cos(90^\circ) =$ _______________

$\cos(330^\circ) =$ ______________ $\cot(180^\circ) =$ _______________

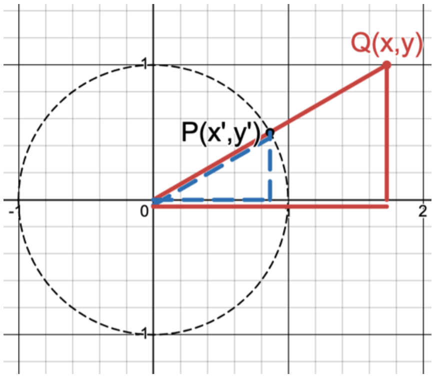
$\tan(225^\circ) =$ ______________ $\sin(-150^\circ) =$ _______________

(Note: if using your calculator, you would need to be in *degree* mode)

Now suppose the angle is created not by considering a point of the unit circle, but by *any* point on the terminal side, which will often be the case when the input is an angle.



How can we find the value of the trig. functions using this new point?



Using similar triangles, we find

$\sin(\theta) =$ ______________ $\csc(\theta) =$ ______________

$\cos(\theta) =$ ______________ $\sec(\theta) =$ ______________

$\tan(\theta) =$ ______________ $\cot(\theta) =$ ______________

Example: Given that the point(-2, 3) is on the terminal side of angle θ , find the values for all the trig functions of θ . *(Note:we aren't given the angle here. Sometimes we are given the angle, sometimes we are just given information* ***about*** *the angle)*

First compute r = ____________________

$\sin(\theta) =$ ______________ $\csc(\theta) =$ ______________

$\cos(\theta) =$ ______________ $\sec(\theta) =$ ______________

$\tan(\theta) =$ ______________ $\cot(\theta) =$ ______________

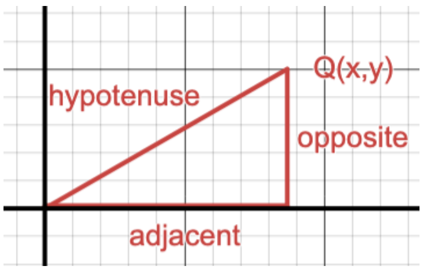
Much of the material in this section is a repeat of material we have already covered, but these new definitions give us another tool for working some problems we did previously.

<u>Examples: Finding All Trig Values Given the Value of One of Them</u>

Given that $\cos(\theta) = -\frac{3}{5}$ and θ is in Quadrant **III**, find the values of the other 5 trig functions of θ

Given that $\tan(\theta) = -4$ and $\sin(\theta) > 0$, find the values of the other 5 trig functions as θ

6.2 Trigonometry in Right Triangles

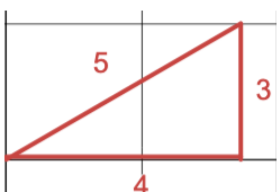


$\sin(\theta) = \frac{y}{r} =$ ______________ $\qquad \csc(\theta) = \frac{1}{\sin(\theta)} = \frac{hyp}{opp}$

$\cos(\theta) = \frac{x}{r} =$ ______________ $\qquad \sec(\theta) = \frac{1}{\cos(\theta)} = \frac{hyp}{adj}$

$\tan(\theta) = \frac{y}{x} =$ ______________ $\qquad \cot(\theta) = \frac{1}{\tan(\theta)} = \frac{adj}{opp}$

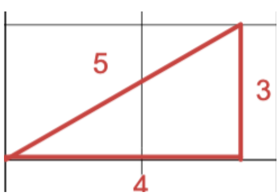
Example: Given the following triangle, find the values of the 6 trig functions. *(Note:we aren't given the angle here)*



$\sin(\theta) =$ ______________ $\qquad \csc(\theta) =$ ______________

$\cos(\theta) =$ ______________ $\qquad \sec(\theta) =$ ______________

$\tan(\theta) =$ ______________ $\qquad \cot(\theta) =$ ______________



$\sin(\alpha) =$ ______________ $\qquad \csc(\alpha) =$ ______________

$\cos(\alpha) =$ ______________ $\qquad \sec(\alpha) =$ ______________

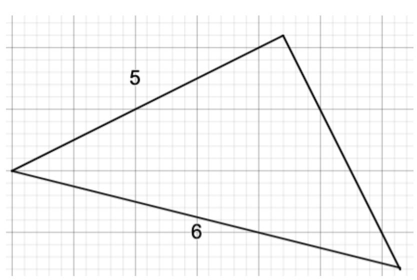
$\tan(\alpha) =$ ______________ $\qquad \cot(\alpha) =$ ______________

Example: Given the following triangle, find the values

$\sin(\theta) =$ ______________

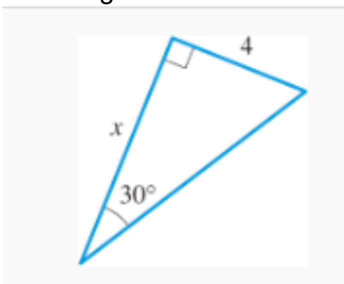
$\cos(\theta) =$ ______________

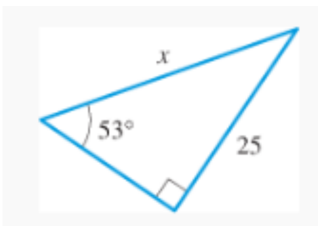
$\tan(\theta) =$ ______________



Applications of Right Triangle Trig

"Solving a Right Triangle". Given certain information about a right triangle, find all remaining sides/angles.

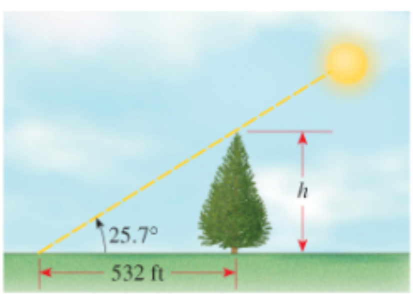




Angle of Elevation/Depression

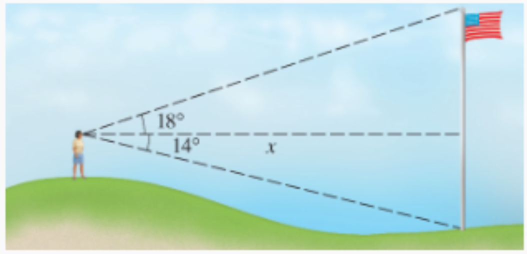
Examples:

A giant redwood tree casts a shadow 532 ft long. Find the height of the tree if the angle of elevation of the sun is 25.7°.



56. **Distance at Sea** From the top of a 200-ft lighthouse, the angle of depression to a ship in the ocea is 23°. How far is the ship from the base of the lighthouse?

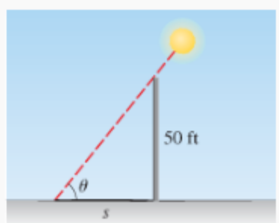
60. **Determining a Distance** A woman standing on a hill sees a flagpole that she knows is 60 ft tall. The angle of depression to the bottom of the pole is 14°, and the angle of elevation to the top of the pole is 18°. Find her distance x from the pole.



6.4 More Right Triangle Problems, where Inverse Trig Functions Are Used

40. **Angle of the Sun** A 96-ft tree casts a shadow that is 120 ft long. What is the angle of elevation of the sun?

42. **Height of a Pole** A 50-ft pole casts a shadow as shown in the figure.



(a) Express the angle of elevation θ of the sun as a function of the length s of the shadow.

(b) Find the angle θ of elevation of the sun when the shadow is 20 ft long.

Examples: Using Right Triangles to Find All Trig Values Given the Value of One of Them

Given that $\cos(\theta) = -\frac{3}{5}$ and θ is in Quadrant III, find the values of the other 5 trig functions of θ

Given that $\tan(\theta) = -4$ and $\sin(\theta) > 0$, find the values of the other 5 trig functions as θ

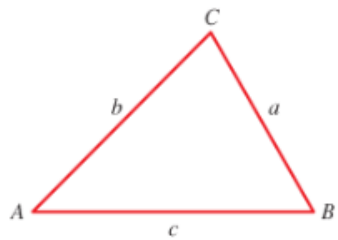
Mixed Compositions

$\tan\left(\sin^{-1}\left(\frac{4}{5}\right)\right)$

$\sec\left(\tan^{-1}\left(\frac{1}{4}\right)\right)$

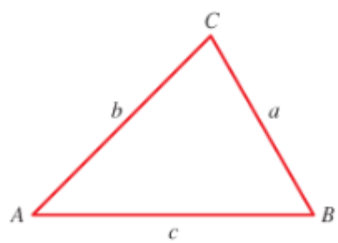
6.5 Law of Sines

The next two sections discuss how we can “solve” (find missing parts) of _________________(non-right) triangles.



12.1 Law of Sines

If we create right triangles by dropping a perpendicular from C to the side AB, we can use what we know about right triangles to find parts of triangle ABC.

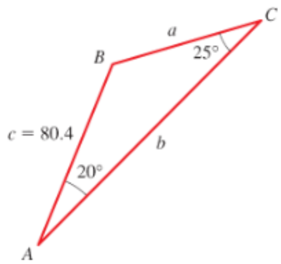


Law of Sines

$\frac{\sin A}{A} = \frac{\sin B}{B} = \frac{\sin C}{C}$ which can also be written. $\frac{A}{\sin A} = \frac{B}{\sin B} = \frac{C}{\sin C}$

Law of Sines Examples

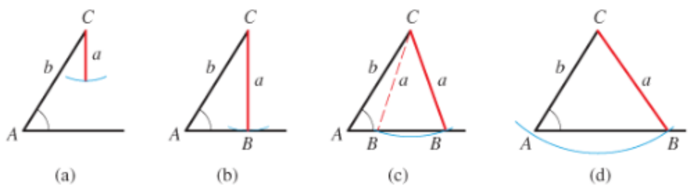
Example 2 (book): Find the remaining parts:



Example: $A = 30^\circ$, $a = 1$, $c = 4$

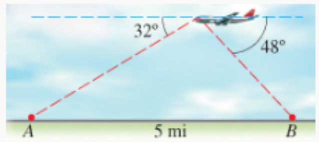
Example: $A = 30^{\circ},\ a = 3, \quad c = 4$

The ambiguous case



Applications

32. **Flight of a Plane** A pilot is flying over a straight highway. He determines the angles of depression to two mileposts, 5 mi apart, to be $32°$ and $48°$, as shown in the figure.

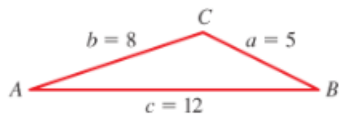


(a) Find the distance of the plane from point A.

(b) Find the elevation of the plane.

6.5 Law of Cosines

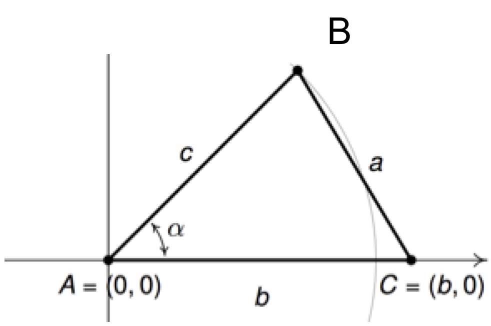
Example: Use the Law of Sines to find the remaining parts of the triangle shown



Development of the Law of Cosines

Can we find a relationship relating the sides of an oblique triangle? Suppose we superimpose a coordinate system onto a general triangle as shown.

What are the coordinates of point B? ______________________________

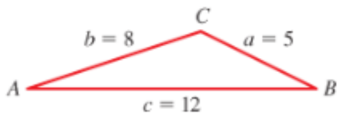


Law of Cosines

$$a^2 = b^2 + c^2 - 2ab\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$

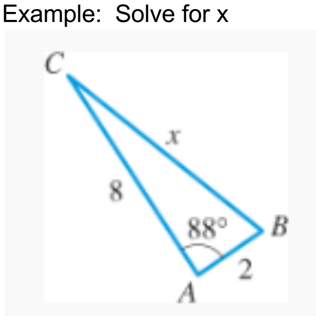
$$c^2 = \_\_\_\_^2 + \_\_\_\_^2 - 2\_\_\_\_\_\_\cos\_\_\_\_$$

Example: Find all the remaining parts

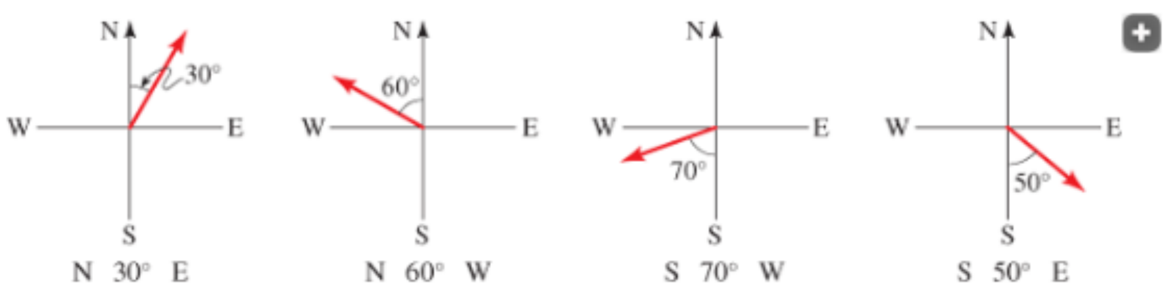


Tip: It is helpful to find the ________________________first. (If we find the largest angle first, the others must be acute)

Example: Solve for x



Navigation (Bearing).



Example:

44. **Navigation** Two boats leave the same port at the same time. One travels at a speed of 30 mi/h in the direction N 50° E, and the other travels at a speed of 26 mi/h in a direction S 70° E (see the figure). How far apart are the two boats after 1 h?